

# Optimum Decision Policy For Replacement of Conventional Energy Sources by Renewable Ones

Maryam Parsa<sup>1</sup>, NasirUddin Ahmed<sup>\*2</sup>, Mustapha C.E. Yagoub<sup>3</sup>

School of Electrical Engineering and Computer Science, University of Ottawa, ON, Canada

<sup>1</sup>mparsa@uottawa.ca; <sup>\*2</sup>ahmed@site.uottawa.ca; <sup>3</sup>myagoub@site.uottawa.ca

## Abstract

With the increase of world population and industrial growth of developing countries, demand for energy, in particular electric power, has gone up at an unprecedented rate over the last four decades. To meet the demand, electric power generation by use of fossil fuel has increased enormously thereby producing increased quantity of greenhouse gases contributing more and more to atmospheric pollution which, climate Scientists believe, can adversely affect the global climate, and health and welfare of world population. In view of these, there is global awareness of looking for alternate sources of energy such as natural gas, hydropower, wind, solar, geothermal and finally biomass etc. It is recognized that this requires replacement of existing infrastructure with new systems, which cannot be achieved overnight. Optimal control theory has been widely used for the last five decades in diverse areas of physical sciences, medicine, engineering, economics and social sciences.

The main motivation of this article is to use this theory to find the optimum strategy (decision or policy) for integration of all currently available renewable energy sources with the existing electric power generating systems with the ultimate goal of elimination of fossil fuel. Eight main energy sources such as Coal, Petroleum, Natural Gas, Conventional Hydro, Wind, Solar, Geothermal and Biomass are considered in a dynamic model. The state of the dynamic model represents the level of power generation from each of the sources at any time  $t$ . The proposed objective function is based on the desired target level of power generation from each of the available sources at the end of the plan period while reducing the production of greenhouse gasses. Pontryagin Minimum principle is used to determine the optimal control or decision policy. Official released data from the U.S. Energy Information Administration is used as a case study. Based on this data and a mathematical model proposed in a paper (Miah, Ahmed and Chowdhury, 2012) published by Energy Economics combined with the minimum principle, an optimal policy is presented for integration of renewable energy sources to the national power grid.

## Keywords

*Optimization; Mathematical Models; Optimal Control; Optimum*

*Decision Policy; Conventional and Renewable Energy Sources*

## Introduction

With the increase of world population and unprecedented industrial growth in many populous countries, demand for energy, in particular electricity, has been steadily increasing over last 30-40 years. To meet this ever increasing demand, use of fossil fuel for generation of electricity has been rising and along with that the production of toxic gasses potentially threatening the climate and public health. On the other hand, the conventional energy sources such as Coal and Petroleum are becoming more and more scarce although there are scientists who believe that there are still ample supply of these sources (Shafiee, Topal, 2009). Most of the researchers think that these supplies are transient and will be depleted within next sixty years (Steinberger, 2009). In any case, considering just the adverse effects of pollution on health and global climate, it has become mandatory for industrial countries to seek for alternate sources and follow a policy of gradual replacement of polluting power sources by clean and renewable ones without adversely affecting the economy.

In this paper, we consider all eight main energy sources used for generation of electricity. These are Coal, Petroleum, Natural Gas, Hydro, Wind, Solar, Wood and Biomass and Geothermal. Our choice of these sources is based on the availability of comprehensive official released data of U.S. Energy Information Administration (U.S. EIA). This data is used along with optimal control (decision) theory (Ahmed, 1988) to develop a methodology for optimal reduction or replacement of fossil fuel by renewable energy sources.

In a recent paper (Miah, Ahmed and Chowdhury, 2012) Lotka-Volterra model was proposed for use as a dynamic model of power generation involving only

two sources such as Renewable and Conventional sources. We use a similar model but for eight different sources and use the available official data (U.S. EIA) to validate the model and then develop optimal generation policy based on the Pontryagin minimum principle (Ahmed, 1988). The Lotka-Volterra system of nonlinear differential equations (Ahmed, 1988, p24) can be used for modeling cooperative as well as competing agents in an ecological environment as well as in market economy.

Rest of the paper is organized as follows. The dynamic model of the system and the proposed cost functional are presented in section II. This is followed by the system identification problem (also called inverse problem) and the problem concerning optimum decision policy in sections III and IV respectively. Different optimization problems are discussed in section V. Numerical algorithms and data are presented in sections VI and VII respectively. The numerical results are presented in section VIII. The paper is concluded in section IX.

## Problem Description and Dynamic Model of the System

### Lotka-Volterra Model

The dynamics of biological ecosystems are basically described by a system of Lotka-Volterra equations (Ahmed, 1988, p24), (Goel, Maitra and Montroll, 1971). Basically this model can be used in both cooperative and competitive environment. More recently, this model has been used also in the study of energy economics (Miah, Ahmed and Chowdhury, 2012). In the study of energy economics, there are many competing sources for power generation and hence this model can be suitably modified to model the energy generation market environment.

For the problem considered in this article, a system of eight first-order nonlinear differential equations is introduced as follows:

$$\dot{x}_i = u_i x_i - \sum_{j=1, j \neq i}^8 \beta_{ij} x_i x_j \quad (1)$$

where  $x_i$ ,  $i = 1 \dots 8$  denotes the level of power generated by the  $i^{\text{th}}$  source at any given time. In particular,  $x_1(t)$  is the level of electric power generated from *Coal* at time  $t$ ,  $x_2(t)$  is the level of electricity generated from *Petroleum*,  $x_3(t)$  is the level of electricity generated from *Natural Gas*,  $x_4(t)$  is the level of electricity generated from *Conventional Hydro*,  $x_5(t)$  is the level of electricity generated from *Wind*,  $x_6(t)$  is the

level of electricity generated from *Solar Power*, (including thermal and photovoltaic),  $x_7(t)$  is the level generated from *Geothermal Source* and finally  $x_8(t)$  is the level of generation from *Wood and Biomass*. The vector

$$x(t) \equiv (x_1(t), \dots, x_8(t))', t \geq 0$$

denotes the state (level) of power generation from each of the sources as a function of time.

In the equation (1),  $u_i$ ,  $i = 1 \dots 8$  represents the percentage increase or decrease of generation from the  $i^{\text{th}}$  source. The parameter  $\beta_{ij}$  describes the impact of generation level of the  $j^{\text{th}}$  power source on the growth (or decline) rate of generation level of the  $i^{\text{th}}$  power source and similarly  $\beta_{ji}$  is the impact of the generation level of the  $i^{\text{th}}$  power source on the growth (decline) rate of generation level of the  $j^{\text{th}}$  energy source. Note that in general  $\beta_{ij} \neq \beta_{ji}$ . In other words, the matrix of interaction is not necessarily symmetric. There are fifty-six interaction parameters in the model proposed for our system and they must be identified before undertaking policy optimization. This will be considered in the "System Identification" section.

### Performance or Cost Function

The primary goal of this paper is to develop a methodology for gradual and optimal replacement of the polluting energy sources, such as Coal and Petroleum, by clean and renewable energy sources, such as Natural Gas, Wind, Solar, Geothermal and Biomass. The objective function to be chosen must incorporate the main concerns of the planner. For example, one may be interested to meet the target level of generation at the end of the plan period while reducing the cost of implementation and greenhouse gas emission during the plan period.

The following general expression given by equation (2), called the cost function, can be used to include many such objectives.

$$J(u) = \int_0^{t_f} \ell(t, x(t), u(t)) dt + \Phi(x(t_f)) \quad (2)$$

The cost function given by equation (2) consists of two parts representing the running cost and the terminal cost. The running cost may include the implementation cost and the penalty for producing greenhouse gasses during the plan. The terminal cost represents the mismatch between the desired level of generation and the actual level of generation reached by the system at the end of the plan period.

The objective is to determine the decision (or control) policy  $u$  as a function of time, taking values from the

set  $U=\{u: |u_i| \leq 1, i = 1 \dots 8\}$  so that it minimizes the cost function  $J(u)$ . To determine the optimum decision policy, we need the relevant mathematical theory of optimal control and system identification presented in the following sections.

### Pontryagin Minimum Principle and Optimal Control Theory

Pontryagin Minimum Principle is a powerful tool for dynamic optimization and so it is clearly suitable for the proposed problem of this article. For detailed theory see (Ahmed 1988, Kirk 2004). According to (Ahmed 1988, Kirk 2004, Moss and Kwoka 2010, Benson and Franklin 2008) the Hamiltonian function plays an important role in this method of optimization.

Consider a general n-dimensional system with state equation (n = 8 in our problem)

$$\dot{x} = f(t, x, u), x(0) = x_0 \quad (3)$$

and the objective functional (2) in its general form. The Hamiltonian function: ( $\cdot$  means dot product) is given by

$$H(t, x, \psi, u) \equiv \langle f(t, x, u), \psi \rangle + \ell(t, x, u) \quad (4)$$

The costate equation is

$$\dot{\psi} = -H_x = -f_x^T(t, x, u)\psi - \ell_x(t, x, u) \quad (5)$$

where  $f_x$  is the Jacobian matrix of the vector  $f$  and  $f_x^T$  is its transpose. The boundary conditions are as follows:

$$x(0) = x_0, \psi(t_f) = \Phi_x^T(x(t_f)) \quad (6)$$

using the state Equation (3), costate Equation (5) and the Hamiltonian function Equation (4), optimum decision policy is determined to minimize the cost function Equation (2).

### Necessary Conditions of Optimality

Since we are mainly interested in application, we state the necessary conditions of optimality without proof. For details see (Ahmed 1988, Kirk 2004, Benson and Franklin 2008). They are summarized as follows:

In order for a (control/decision) policy  $u^o$  over a plan period  $I \equiv [0, t_f]$  to be optimal, it is necessary to have a co-state  $\psi^o$  corresponding to the policy  $u^o$  and the associated state process  $x^o$  satisfying the inequality (7), given below and the system of equations (8) and (9) subject to the boundary conditions (6):

$$H(t, x^o(t), \psi^o(t), u^o(t)) \leq H(t, x^o(t), \psi^o(t), u) \quad (7)$$

$$\dot{x}^o = H_{\psi}(t, x^o(t), \psi^o(t), u^o(t)) = f(t, x^o(t), u^o(t)) \quad (8)$$

$$\begin{aligned} \dot{\psi}^o &= -H_x(t, x^o(t), \psi^o(t), u^o(t)) \\ &= -f_x^T(t, x^o(t), \psi^o(t), u^o(t)) - \ell_x(t, x^o(t), u^o(t)) \end{aligned} \quad (9)$$

The optimum solution of the problem considered in this article, is contained in the family of functions  $\{u^o, x^o, \psi^o\}$  satisfying all the above equations called the extremals. Later in the "Numerical Algorithm" section, the way these equations are used is discussed.

### System Identification Problem

As stated before, there are fifty six unknown parameters in the model chosen for this problem Equation (1) that provides the interaction (coupling) between generation levels of different types of power sources. It is interesting to note that optimal control theory can be used also for identification of parameters ( $\beta \equiv \beta_{ij}, 1 \leq i, j \leq 8$ ). Given that  $y(t), t \in I$ , is the natural response of the system, and  $x(t, \beta), t \in I$ , the response of the proposed system model (1), a measure of mismatch can be introduced by the following functional

$$J(\beta) = \frac{1}{2} \int_0^{t_f} |x(t, \beta) - y(t)|^2 dt \quad (10)$$

where  $|\cdot|$  denotes the Euclidean norm in  $R^{[8]}$ . The problem is to determine the parameter  $\beta^o$  that minimizes the identification error as defined by Equation (10).

Historical data for all eight power sources are available and can be used for  $y(t), t \in I$  in the expression (10). As stated in (Ahmed, 1988), system identification using optimal control theory approach is possible through the necessary conditions, as indicated below.

The Hamiltonian function for the system identification is given by the following expression

$$H(t, x, \psi, \beta) = \langle f(t, x, \beta), \psi \rangle + \frac{1}{2} |x(t, \beta) - y(t)|^2 \quad (11)$$

where  $\langle \cdot, \cdot \rangle$  means dot product. The corresponding necessary conditions of optimality can be stated as follows: For  $\beta^o$  to be the best parameter, it is necessary that the triple  $\{\beta^o, x^o, \psi^o\}$  satisfies the following set of equations and inequality

$$\dot{x} = f(t, x^o, \beta^o), x^o(0) = x_0 \quad (12)$$

$$\dot{\psi}^o = -f_x^T(t, x^o, \beta^o)\psi^o - (y(t) - x^o(t)), \psi(t_f) = 0 \quad (13)$$

$$\int_{t_0}^{t_f} H(t, x^o(t), \psi^o(t), \beta^o) dt \leq \int_{t_0}^{t_f} H(t, x^o(t), \psi^o(t), \beta) dt \quad (14)$$

All the above equations will be used in the "Numerical Algorithm" section to find the unknown parameters of the system. Once these parameters have been determined, we have a dynamic model for the energy production system. The planner can then use this

model and find the optimal power generation strategy for each of the power sources mentioned above.

### Energy Production Policies

Once the parameter identification problem is solved, one can focus on the energy production planning. For this, different types of cost functions are considered which reflect the short term and long term social needs. For the purpose of this paper, three main problems are considered. The first one aims at reaching the final desired generation levels for each of the power source. The second one also is concerned with reaching the goal while keeping the implementation cost to a minimum. The third problem is concerned with minimizing the usage of polluting power sources while reaching the total demand at the end of the plan period.

#### (P1) Optimum Decision Policy for Meeting the Target

Here the primary objective is to meet the production goal at the end of the plan period. For this, the cost functional is given by

$$J(u) = \Phi(x(t_f)) \equiv \frac{1}{2} \left\{ \sum_{i=1}^8 \delta_i (x_i(t_f) - x_i^d)^2 \right\} \quad (15)$$

where  $\{x_i^d\}$  are the desired target levels of generation from each of the sources considered and  $\{x_i(t_f)\}$  are the actual generation levels reached at the end of the plan period. The objective is to find an optimal policy  $u^o$  for the system (1) that minimizes this functional.

#### (P2) Optimum Policy for Meeting the Target with Implementation Cost

Given the current levels of power generation from each of the available sources, and the plan period, the objective is to reach a prespecified target level of generation while keeping the implementation cost as low as possible. For this, one must use the terminal cost function including the implementation cost. This is given by the following expression

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2(t) \right\} dt + \frac{1}{2} \left\{ \sum_{i=1}^8 \delta_i (x_i(t_f) - x_i^d)^2 \right\} \quad (16)$$

where  $\{q_i, \delta_i \geq 0\}$  are suitable weights. The planner can choose the weights in the cost function as desired.

#### (P3) Optimum Policy for Meeting total Demand with Implementation and Environmental Cost

Among the primary power sources chosen in this article, Coal and Petroleum are the most polluting ones. Natural gas and the renewable sources are clean. For this problem, we need the relationship between

the level of electric power generation and the corresponding level of production of greenhouse gasses for each of the sources. This relation is well known to engineers and it is quite reasonable to assume that it is given by an expression of the form:

$$F(x) = v|x|^q, q \geq 1, v > 0 \quad (17)$$

Only by experiment, can engineers determine the two parameters  $\{v, q\}$  for each of the conventional sources. It is known that petroleum produces 1.23 times more CO<sub>2</sub> than coal for the same level of electric power generation (CO<sub>2</sub>, EIA, 2012). This relationship is considered as a reference in our problem for numerical results. For numerical purpose, we choose  $q = 2$ . The objective functional for this problem is given by

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2(t) + \sum_{i=1}^8 w_i x_i^2(t) \right\} dt + \frac{1}{2} \Gamma (X^d(t_f) - \sum_{i=1}^8 x_i(t_f))^2 \quad (18)$$

where the first sum represents the implementation cost and the second sum represents the penalty for environmental damage due to usage of coal and petroleum. The third term represents the mismatch between the total energy demand  $X^d$  and the sum of the energies produced from each of the sources.

### Numerical Algorithm

We consider the three optimization problems given by Equations (15), (16) and (18). Before considering the optimization problems, we must identify the system parameters  $\{\beta \equiv \beta_{ij}\}$ . We use gradient technique along with optimal control theory to determine the fifty six unknown parameters  $\{\beta \equiv \beta_{ij}\}$  based on the U.S energy data. Then these parameters are fixed and the model is set. Next, we use the cost functionals (15), (16) and then (18) to determine the optimum decision policy for integrating the renewable power sources into the electricity power generation (Miah, Ahmed and Chowdhury 2012, Ahmed 1988) based on MATLAB software (Wang, 2009).

#### Methodology for System Identification

Step 1 Subdivide the plan period into  $N$  equal subinterval.

Step 2 Integrate the state equation (12) for the plan period with any assumed  $\beta \equiv \beta_{ij}$  and the given initial conditions at  $t_0$ . Let  $\{\beta_n, x_n\}$  denote the values of  $\beta$  and the state  $x$  at  $n^{th}$  iteration.

Step 3 Integrate the costate equation (13) backward in time with the pair  $\{\beta_n, x_n\}$  replacing  $\{\beta^0, x^0\}$  with the

terminal conditions at  $t_f$  as given in equation (13). This will give the costate  $\psi_n$ .

Step 4 Determine the gradient vector using the data  $\{\beta_n, x_n, \psi_n\}$  as follows:

$$g_n = \int_{t_0}^{t_f} \frac{\partial H}{\partial \beta}(t, x_n, \psi_n, \beta_n) dt = \int_{t_0}^{t_f} f_{\beta}^T(t, x_n, \beta_n) \psi_n dt \quad (19)$$

Step 5 Determine if the inequality (14) is satisfied with  $\{\beta_n, x_n, \psi_n\}$  replacing  $\{\beta^0, x^0, \psi^0\}$ . Stop the iteration if it is satisfied (because the triple  $\{\beta_n, x_n, \psi_n\}$  is optimal). If not, check the inequality

$$\int_{t_0}^{t_f} \|H_{\beta}(x_n(t), \psi_n(t), \beta_n)\|^2 dt \leq \delta \quad (20)$$

satisfied. Stop the iteration if it is satisfied. If not, replace  $\beta_n$  for next iteration using  $\beta_{n+1}$  given by

$$\beta_{n+1} = \beta_n - \varepsilon g_n \quad (21)$$

and continue the search from Step 2.

### Methodology for Computation of Optimum Decision Policy

Here also we choose the gradient technique. Thus the procedures is similar to that for parameter identification with the following modifications:

- The parameter  $\beta_n$  is replaced by  $u_n(t)$ ,  $t \in I$  which denotes the decision policy at the  $n^{th}$  iteration
- Equation (19) is replaced by

$$g_n(t) = H_u(x_n(t), \psi_n(t), u_n(t)) \quad (22)$$

- Inequality (14) is replaced by equation (23)

$$H(x_n(t), \psi_n(t), u_{n+1}(t)) = H(x_n(t), \psi_n(t), u_n(t)) - \varepsilon \|H_u(x_n(t), \psi_n(t), u_n(t))\|^2 + o(\varepsilon) \quad (23)$$

### Numerical Data

Increasing energy demand and environmental concerns must be addressed in a way so that demand is met while impacts on environment is kept to a minimum. With this goal in mind, we have chosen the available data from U.S. Energy Information Administration (EIA) website (U.S. EIA). For system identification concerned with the determination of the unknown parameters of the model, historical data for all the chosen energy sources are taken from the latest released data of EIA website (Annual Energy Outlook, 2012). This is given in Table 1.

After determining the unknown parameters of the system model using the historical data of Table 1, one can focus on optimization based on this model. For

optimum energy policy, it is necessary to define the desired values for each of the power sources. Based on the "20% Wind Energy by 2030" scenario released by (NREL, 2008), the generation level of Wind power (source) should cover around 20% of the total electricity generation by year 2030. In the case of Solar power source (EERE, 2008), providing 10% of the total electricity generation by year 2025 is the possible desired value. Finally according to (C2ES, 2012), 223 gigawatts of new generating capacity is planned to be added to the electric power industry of the U.S. This is based on the increasing population and electricity demand of the country. Natural Gas power source stands for 60% of this increased capacity, while Coal power source is responsible for 11%.

TABLE 1 GENERATION LEVEL OF ELECTRIC POWER SECTORS (TRILLION KILOWATTHOURS)- HISTORICAL DATA (ANNUAL ENERGY OUTLOOK, 2012)

	Year 2010	Year 2015
Coal	1.831	1.562
Petroleum	0.034	0.026
Natural Gas	0.898	1.028
Conventional Hydropower	0.25532	0.29543
Wind	0.09449	0.15097
Solar	0.00128	0.00647
Geothermal	0.01567	0.01868
Wood and Biomass	0.01151	0.02128

In this paper, the target (or desired) values are based on the latest data released by (Annual Energy Outlook, 2012) for "Low Renewable Technology Cost Case" shown in Table 2. However, our procedure can be used for any desired target. All the data presented here will be used in the following section for system identification and then this model is used to generate the optimum decision policy.

TABLE 2 GENERATION LEVEL OF ELECTRIC POWER SECTORS (TRILLION KILOWATTHOURS)- PROJECTION BASED ON "LOW RENEWABLE TECHNOLOGY COST CASE" (ANNUAL ENERGY OUTLOOK, 2012)

	Year 2015	Year 2035
Coal	1.562	1.780
Petroleum	0.026	0.028
Natural Gas	1.028	1.037
Conventional Hydropower	0.29543	0.32178
Wind	0.15097	0.31055
Solar	0.00647	0.0869
Geothermal	0.01868	0.05089
Wood and Biomass	0.02128	0.07841

### Numerical Results

The simulation results obtained by the use of MATLAB software using the numerical algorithm mentioned in section VI are presented here. The first

part deals with the problem of system identification using the U.S. energy projection data over six year period. This gives the unknown parameters  $\beta \equiv \{\beta_{ij}\}$  including  $u \equiv \{u_i\}$ . We use the identified parameters  $\beta \equiv \{\beta_{ij}\}$  for the second part dealing with optimization. Here three different scenario are taken into account giving the optimum decision policy for each of them.

### System Identification

The purpose of this section is to determine the fifty six unknown parameters of the system based on the historical data of Table 1. This is achieved by minimizing the (error) functional given by the expression (10). Minimizing this functional is equivalent to minimizing the gap between the model response and the actual data of the Table 1. The results are shown in Table 3 and Table 4. Figure 1 shows the identification error as a function of iteration. After 10000 iterations, the error reduces to almost zero. Figure 2 shows a sample of parameters as function of iteration. It is clear from the figure that they converge to constant values.

TABLE 3 COMPUTED VALUES OF PARAMETERS AFTER 10,000 ITERATIONS

$\beta_{ij}$	Value	$\beta_{ij}$	Value	$\beta_{ij}$	Value	$\beta_{ij}$	Value
$\beta_{12}$	0.00226	$\beta_{23}$	0.00495	$\beta_{35}$	0.00676	$\beta_{48}$	0.00024
$\beta_{21}$	0.00971	$\beta_{32}$	0.00109	$\beta_{53}$	0.08901	$\beta_{84}$	0.02516
$\beta_{13}$	0.02771	$\beta_{24}$	0.00356	$\beta_{36}$	3.55e-6	$\beta_{56}$	0.00010
$\beta_{31}$	0.02508	$\beta_{42}$	0.00072	$\beta_{63}$	0.03664	$\beta_{65}$	0.00385
$\beta_{14}$	0.00944	$\beta_{25}$	0.00046	$\beta_{37}$	4.35e-5	$\beta_{57}$	0.00027
$\beta_{41}$	0.03931	$\beta_{52}$	0.00176	$\beta_{73}$	0.00750	$\beta_{75}$	0.00078
$\beta_{15}$	0.00460	$\beta_{26}$	9.28e-5	$\beta_{38}$	3.20e-5	$\beta_{58}$	0.00093
$\beta_{51}$	0.14887	$\beta_{62}$	0.00238	$\beta_{83}$	0.08850	$\beta_{85}$	0.00931
$\beta_{16}$	0.00089	$\beta_{27}$	1.20e-5	$\beta_{45}$	0.00202	$\beta_{67}$	0.00063
$\beta_{61}$	0.07470	$\beta_{72}$	0.00038	$\beta_{54}$	0.02075	$\beta_{76}$	1.07e-5
$\beta_{17}$	0.00073	$\beta_{28}$	3.53e-5	$\beta_{46}$	2.75e-5	$\beta_{68}$	0.00046
$\beta_{71}$	0.01529	$\beta_{82}$	0.00735	$\beta_{64}$	0.01041	$\beta_{86}$	0.00012
$\beta_{18}$	0.00407	$\beta_{34}$	0.00070	$\beta_{47}$	0.00033	$\beta_{78}$	0.00009
$\beta_{81}$	0.18045	$\beta_{43}$	0.01928	$\beta_{74}$	0.00213	$\beta_{87}$	0.00054

TABLE 4 COMPUTED VALUES OF PARAMETERS AFTER 10,000 ITERATIONS

$u_i$	Value	$u_i$	Value	$u_i$	Value	$u_i$	Value
$u_1$	-0.019	$u_3$	0.097	$u_5$	0.418	$u_7$	0.191
$u_2$	-0.194	$u_4$	0.128	$u_6$	0.959	$u_8$	0.701

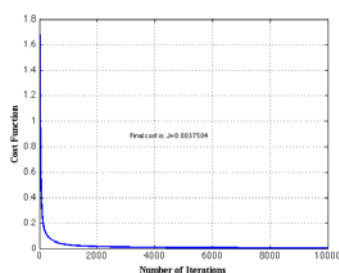
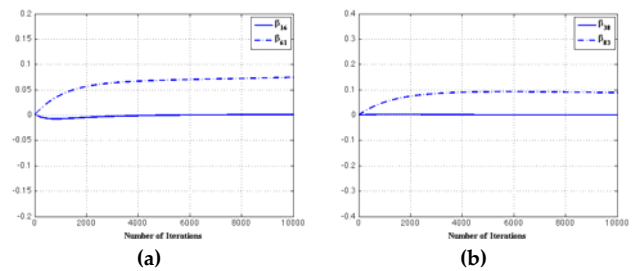


FIG. 1 IDENTIFICATION ERROR AFTER 10,000 ITERATION

FIG. 2 CALCULATED VALUES FOR (a)  $B_{16}$ ,  $B_{61}$  AND (b)  $B_{38}$ ,  $B_{83}$  AFTER 10,000 ITERATIONS

### (P1) Optimum Decision Policy for Meeting the Target

Here the cost functional is given by equation (15) as reproduced below

$$J(u) = \Phi(x(t_f)) \equiv \frac{1}{2} \left\{ \sum_{i=1}^8 \delta_i (x_i(t_f) - x_i^d)^2 \right\} \quad (24)$$

where the values of  $\delta_i$  are shown in Table 5.

TABLE 5 ASSUMED VALUES FOR WEIGHTS IN EQUATION (24) FOR (P1)

$\delta_i$	Value	$\delta_i$	Value	$\delta_i$	Value	$\delta_i$	Value
$\delta_1$	8	$\delta_3$	3	$\delta_5$	4	$\delta_7$	5
$\delta_2$	5	$\delta_4$	3	$\delta_6$	20	$\delta_8$	10

The objective is to find an optimal policy  $u^o$  for the system (1) that minimizes this functional. For this problem initial values are taken from the data shown in Table 2 for the year 2015. The data for the desired values are based on the projection of the U.S. Energy Information Administration presented also in (Annual Energy Outlook, 2012) and shown in Table 2 for the year 2035. Using the methodology given in section VI, the optimal policy is computed as shown in Figures 3-5. Figure 3 shows the optimal path of generation level from each of the considered sources. Figure 4 shows the optimal decision (i.e. generating) policies corresponding to all major energy sources in the United States. Figure 5 shows the convergence of the cost function with increasing iteration. It is clear from the Figure 3 that the target set by U.S. EIA is met very closely by following the control policy shown in Figure 4.

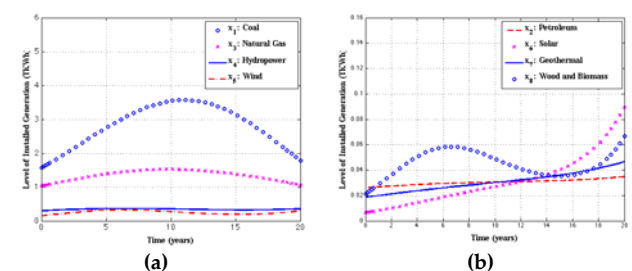


FIG. 3 (P1) STATE TRAJECTORIES FOR THE PLAN PERIOD OF 20 YEARS FOR (a) COAL, NATURAL GAS, HYDRO, WIND AND (b) PETROLEUM, SOLAR, GEOTHERMAL, WOOD AND BIOMASS

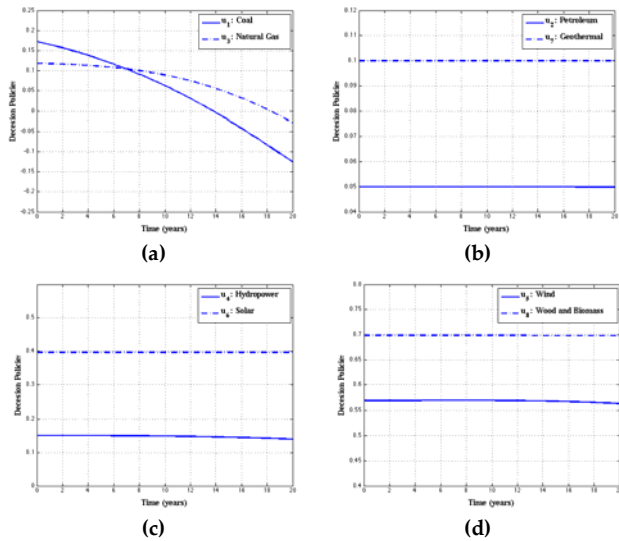


FIG. 4 (P1) DECISION POLICY FOR THE PLAN PERIOD OF 20 YEARS FOR (a) COAL AND NATURAL GAS, (b) PETROLEUM AND GEOTHERMAL, (c) HYDRO AND SOLAR, (d) WIND AND WOOD AND BIOMASS

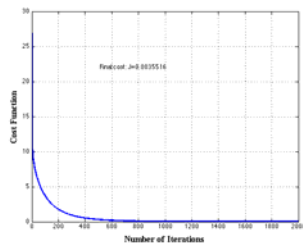


FIG. 5 (P1) MINIMIZED COST FUNCTION AFTER 2,000 ITERATIONS

### (P2) Optimum Policy for Meeting the Target with Implementation Cost

In the previous subsection, it is assumed that the cost of implementation of the control policy is negligible. Here we remove this assumption and introduce the control implementation cost. The implementation cost may involve both capital cost and maintenance cost. The cost functional for this problem is given by the expression (16) as reproduced below:

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2(t) \right\} dt + \frac{1}{2} \left\{ \sum_{i=1}^8 \delta_i (x_i(t_f) - x_i^d)^2 \right\} \quad (25)$$

where  $\{q_i \geq 0, i = 1 \dots 8\}$  are suitable weights representing the unit cost of control policies. It is generally expected that the values for these weights are dependent on the source type and may be comparatively larger for certain sources than others. The values of  $\{q_i\}$  and  $\{\delta_i\}$  are shown in Table 6. Again using the methodology presented in section VI, we find the optimal decision policy as shown in Figures 6-8. Figure 6 shows the path of optimal generation levels (as function of time) for each of the sources and Figure 7 shows the optimal decision (i.e. generating) policies corresponding

to all major energy sources in the United States.

TABLE 6 ASSUMED VALUES FOR WEIGHTS IN EQUATION (25) FOR (P2)

$\delta_i$	Value	$\delta_i$	Value	$q_i$	Value	$q_i$	Value
$\delta_1$	8	$\delta_5$	4	$q_1$	10	$q_5$	1
$\delta_2$	5	$\delta_6$	20	$q_2$	5	$q_6$	0.59
$\delta_3$	3	$\delta_7$	5	$q_3$	3	$q_7$	1.8
$\delta_4$	3	$\delta_8$	10	$q_4$	3	$q_8$	0.68

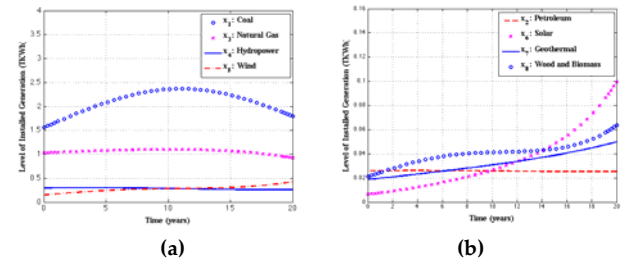


FIG. 6 (P2) STATE TRAJECTORIES FOR THE PLAN PERIOD OF 20 YEARS FOR (a) COAL, NATURAL GAS, HYDRO, WIND AND (b) PETROLEUM, SOLAR, GEOTHERMAL, WOOD AND BIOMASS

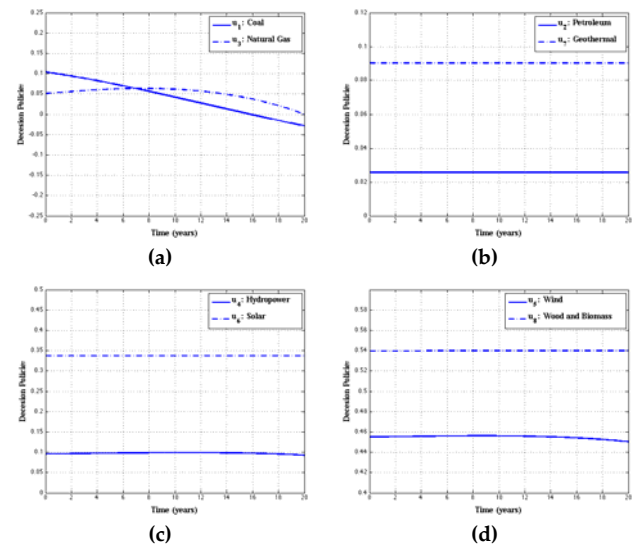


FIG. 7 (P2) DECISION POLICY FOR THE PLAN PERIOD OF 20 YEARS FOR (a) COAL AND NATURAL GAS, (b) PETROLEUM AND GEOTHERMAL, (c) HYDRO AND SOLAR, (d) WIND AND WOOD AND BIOMASS

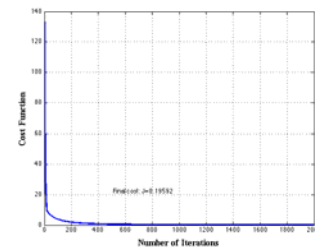


FIG. 8 (P2) MINIMIZED COST FUNCTION AFTER 2,000 ITERATIONS

### (P3) Optimum Policy for Meeting Total Demand with Implementation and Environmental Cost

The objective functional for this problem is given by equation (18) which is reproduced below for easy

reference

$$J(u) = \frac{1}{2} \int_{t_0}^{t_f} \left\{ \sum_{i=1}^8 q_i u_i^2(t) + \sum_{i=1}^8 w_i x_i^2(t) \right\} dt + \frac{1}{2} \Gamma (X^d(t_f) - \sum_{i=1}^8 x_i(t_f))^2 \quad (26)$$

The weights chosen are given in Table 7 and the results are shown in Figures 9-11. Figure 9 shows the level of generation from each of the sources as functions of time. It is clear from the Figure 9 that due to introduction of environmental cost, the usage of polluting sources has been substantially cut down while production from clean sources such as wind, solar and geothermal, biomass has increased sufficiently to meet the demand. Figure 10 shows the optimal decision (i.e. generating) policies corresponding to all major energy sources in US. Convergence of the cost function is shown in Figure 11(a).

Figure 11(b) shows the total electricity generation during the plan period. Electricity demand in the year 2035, as projected by U.S. EIA (Annual Energy Outlook, 2012), is 4.7 TKWh while according to our numerical example it is 5.2045 Trillion kWh. According to this figure, it is observed that the generation level reached at the end of the plan period exceeds the estimated target only by 6.15 percent.

TABLE 7 ASSUMED VALUES FOR WEIGHTS IN EQUATION (26) FOR (P3)

$q_i$	Value	$q_i$	Value	$\omega, \Gamma$	Value
$q_1$	10	$q_5$	1	$\omega_1$	5
$q_2$	5	$q_6$	0.59	$\omega_2$	$5 \times 1.23$
$q_3$	3	$q_7$	1.8	$\Gamma$	10
$q_4$	3	$q_8$	0.68		

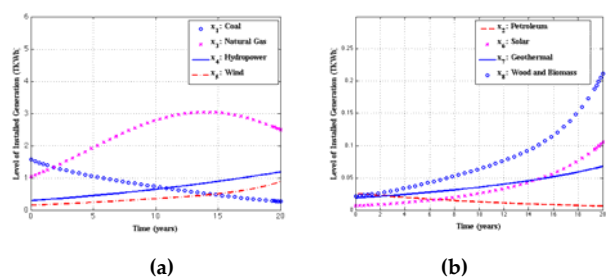


FIG. 9 (P3) STATE TRAJECTORIES FOR THE PLAN PERIOD OF 20 YEARS FOR (a) COAL, NATURAL GAS, HYDRO, WIND AND (b) PETROLEUM, SOLAR, GEOTHERMAL, WOOD AND BIOMASS

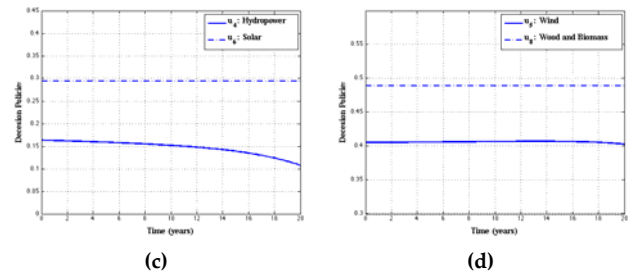
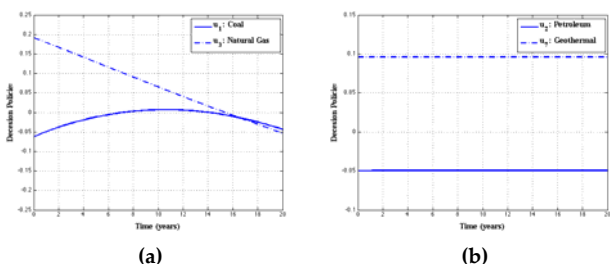


FIG. 10 (P3) DECISION POLICY FOR THE PLAN PERIOD OF 20 YEARS FOR (a) COAL AND NATURAL GAS, (b) PETROLEUM AND GEOTHERMAL, (c) HYDRO AND SOLAR, (d) WIND AND WOOD AND BIOMASS

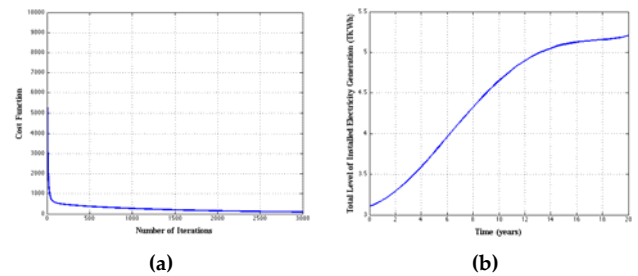


FIG. 11 (P3) (a) MINIMIZED COST FUNCTION AFTER 3,000 ITERATIONS, (b) TOTAL LEVEL OF INSTALLED ELECTRICITY GENERATION DURING THE PLAN PERIOD OF 20 YEARS

## Conclusion

The optimum decision policies based on the official released data from the U.S. Energy Information Administration official website for eight main power sources such as Coal, Petroleum, Natural Gas, Hydropower, Wind, Wood and Biomass, Solar and Geothermal are presented in this paper.

A dynamic model representing the level of electricity generation from each of the selected power sources and the (economic and availability) interactions between them is considered. From optimal control theory, Pontryagin minimum principle is chosen as the appropriate method of optimization. Based on the available historical data, all unknown parameters of the modeled system are identified. For the plan period of 20 years (2015-2035) the optimum decision policies are determined for three different problems.

In the first problem (P1), optimal policy minimizes the gap between the target and the actual levels of generation for all the eight power sources. In the second problem (P2), optimum policy minimizes the gap between the target and the actual levels of generation while keeping the implementation cost as low as possible. In the third problem (P3), the optimum policy tries to satisfy the total demand at the final time while keeping the implementation cost and environmental damage as low as possible.

The procedure used in this article for system identification and optimum decision is completely general and the planner can easily adapt this (optimization) technique to consider any other factor such as investment cost, demands as function of time, and other economic factors by just modifying the cost function and applying the same procedure. The dynamic model used here can be applied to any economic system consisting of cooperating and competing agents in a closed economy. However, the identification and the optimization procedure presented here are applied to any economic sector provided that the dynamic model is chosen appropriately.

## REFERENCES

- Ahmed N.U., "Elements of Finite Dimensional System and Control Theory", Pitman Monographs and Surveys in Pure and Applied Mathematics, vol.37. Longman Scientific and Technical with John Wiley, London, New York, 1988
- Annual Energy Outlook 2012 with projections to 2035, U.S. Energy Information Administration, [http://www.eia.gov/forecasts/aeo/pdf/0383\(2012\).pdf](http://www.eia.gov/forecasts/aeo/pdf/0383(2012).pdf)
- Benson S.M., Franklin M., "Sustainability and energy conversions", MRS Bulletin 33, 2008
- C2ES, Center for Climate and Energy Solutions, "Natural Gas in the U.S. Electric Power Sector", May 2012, <http://www.c2es.org/docUploads/natural-gas-electric-power-sector.pdf>
- CO<sub>2</sub>, EIA, U.S. Energy Information Administration, Environment Analysis and Projections, "U.S. Energy Related Carbon Dioxide Emissions, 2011", August 2012, <http://www.eia.gov/environment/emissions/carbon/>
- EERE, U.S. Department of Energy, Energy Efficiency and Renewable Energy, EERE Network News, "Study: Solar Power Could Provide 10% of U.S. Electricity by 2025", June 2008, [http://apps1.eere.energy.gov/news/news\\_detail.cfm/news\\_id=11835](http://apps1.eere.energy.gov/news/news_detail.cfm/news_id=11835)
- Goel N.S., Maitra S.C., Montroll E.W., "On the Volterra and the non-linear models of interacting populations", Academic Press Inc., 1971
- Kirk D.E., "Optimal control theory, An introduction", Dover publications, Inc., Mineola, New York, 2004
- Miah M.S., Ahmed N.U., Chowdhury M., "Optimum policy for integration of renewable energy sources into the power generation system", Energy Economics 34, 558-567, 2012
- Moss D.L., Kwoka J.E., "Competition policy and the transition to a low-carbon efficient electricity industry", The Electricity Journal 23, Issue 7, 6-15, 2010
- NREL, U.S. Department of Energy, Energy Efficiency and Renewable Energy, "20% Wind Energy by 2030, Increasing Wind Energy's Contribution to U.S. Electricity Supply", July 2008, <http://www.nrel.gov/docs/fy08osti/41869.pdf>
- Shafiee S., Topal E., "When will fossil fuel reserves be diminished?", Energy Policy 37, 181-189, 2009
- Steinberger C.J., "What future for energy and climate?", Presentation in Bad Kissingen, Germany, July 2009
- U.S. EIA, U.S. Energy Information Administration Official Website, [www.eia.gov](http://www.eia.gov)
- Wang X., "Solving optimal control problems with MATLAB-Indirect methods", November 2009